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An Error-Learning Model of Treasury Bill Futures and Implications for the Expectation Hypothesis

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This paper investigates the behavior of expectations of interest rates contained in Treasury bill futures, which has been of increasing interest to financial economists since the expectations are built into investment decisions, portfolio decisions, borrowing and lending decisions, and other financial decisions of firms. The empirical results in this paper reveal that the forecasting error-learning mechanism is a dominant factor for changes in expectations of interest rates reflected in Treasury bill futures contracts. In addition, supporting the expectation hypothesis of the term structure of interest rates, this paper shows that the components of the yield curve are members of a group of rates which are systematically related to each other by the forecasting error and the systematic formation of expectations reflected in interest rate futures.

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I. Introduction

A considerable amount of financial economic research has been devoted to measuring expectations of future interest rate following different approaches ([3], [4], [6], [9], [10], [11], [12], and [13]), since such expectations or changes in expectations have important effects on economic behavior such as investment decisions of firms, portfolio decisions, and borrowing and lending decisions of financial firms. However, the financial futures contracts which were introduced recently have provided us with direct information as to the expected pattern of future spot rates.[1] For example, the yields on Treasury bill futures contracts allow us to observe the pattern of short term interest rates expected by the market to prevail in certain months in the future, utilizing the information about the factors influencing the behavior of interest rate.

Since the market formulates expectations of future interest rates utilizing the information available only at the present time, as the new information becomes available, the market will revise its expectations, which can be identified by shifts in the yield structures of interest rate futures. In particular, if actual market rates differ from what had been anticipated, which has been the most important information set in a wide variety of behavioral contexts of previous studies ([3], [9], [10], [11], and [12]), the market may very well revise its expectations.

Following a previous study [9], this paper investigates the behavior of expectations of interest rates contained in Treasury bill futures,

which has been of increasing interest to practitioners, academicians and others.[2] First, this paper provides operational tests on whether expectations are systematically altered on the basis of errors which have been made in forecasting actual market rates. This test is done by regressing changes in expectations on the forecasting error. In addition, a careful examination of the regression from a different angle provides an indirect test on the expectation hypothesis of the term structure of interest rates.

Section II briefly describes the characteristics of Treasury bill futures (data) and methodology. Section III presents the results and a brief conclusion is contained in Section IV.

II. Data and Methodology

The International Monetary Market of the Chicago Mercantile Exchange began trading on January 6, 1976, in contracts of three-month U.S. Treasury bills for delivery in March, June, September and December. Originally only six contracts were traded: March, June, September and December of 1976, and March and June of 1977. More recently the number of contracts was increased to eight, the latest being for delivery twenty-one months in the future. Each contract calls for delivery of one million dollars (face value) worth of three-month Treasury bills at maturity.[3] Futures trading terminates on the second business day following the three-month Treasury bill auction in the third week of the delivery month. When each contract matures, trading begins in a new contract dated three months beyond the most distinct contract previously traded. Prices on the Treasury bill futures are quoted on a discounted basis. This paper uses spot Treasury bill prices and Treasury bill

futures prices, adjusted to reflect yields, as of Thursday (Wednesday if there is no trading on Thursday) of the third week of each month in order to match as closely as possible the delivery day for Treasury bill futures, for the period of June 1976 - August 1983. Interest rates on three-month spot Treasury bills were gathered from the Wall Street Journal, and interest rates on three-month Treasury bill futures were obtained from the MJK computer tape containing closing prices of Treasury bill futures traded in the International Monetary Market.[4]

First, this paper intends to test whether expectations on interest rates (reflected in Treasury bill futures contracts) are systematically altered on the basis of errors which have been made in forecasting actual market rates.

The hypothesis to be tested can be written as a functional form at time t:[5]

$$ {}_{t+n}F_{t} - {}_{t+n}F_{t-1} = f(R_{t} - {}_{t}F_{t-1}) \tag{1} $$

where F and R represent the expected interest rate for three months (3-month Treasury bill futures) and the actual market rate for three months (3-month spot Treasury bill), respectively. The left-hand side subscript of F represents the maturity month of three-month Treasury bill futures contracts. For example, when n=3 (months) and t is June 1976, the notations are used as follows: ${}_{t+3}F_{t}$ = September 1976 Treasury bill futures observed at June 1976, ${}_{t+3}F_{t-1}$ = September 1976 Treasury bill futures observed at May 1976, R_{t} = spot Treasury bill rate observed at June 1976, and ${}_{t}F_{t-1}$ = June 1976 Treasury bill futures observed at May 1976. Note that $R_{t} - {}_{t}F_{t-1}$ is the forecasting error of

the interest rate for three months, the difference between the actual three-month interest rate at time t and the three-month rate which was expected at time t-1 to prevail at time t. Also, the left hand side of the equation (1), ${}_{t+n}F_t - {}_{t+n}F_{t-1}$, represents changes in expectations on three-month interest rates reflected in Treasury bill futures.

Assuming that the functional relation is linear, it can be expressed as

$$ {}_{t+n}F_t - {}_{t+n}F_{t-1} = \alpha + \beta\,(R_t - {}_tF_{t-1}) \tag{2} $$

Since Treasury bill futures contracts are standardized at three months interval such as March, June, September and December, the above regression will be examined when n equals 3, 6, 9, 12, 15, 18, and 21 months.

In addition, for the purpose of testing the expectations hypothesis of the term structure of interest rates indirectly, the null hypothesis that the dependent variable (changes in expectations) will be synchronized with the independent variable (forecasting error) of the regression (2) will be tested by observing the sign of those variables over time. Note that the conventional expectations hypothesis of the term structure of interest rates asserts that if actual rates are higher than had been anticipated so that the forecasting errors $(R_t - {}_tF_{t-1})$ are positive, the market will revise upward expectations of what interest rates in the future are likely to be, so that changes in expectations $({}_{t+n}F_t - {}_{t+n}F_{t-1})$ are positive. Similarly, if the forecasting errors are negative, the market will systematically revise downward expectations. Also, a careful examination of the intercept term of the

regression (2) will be used to test the significance of the liquidity premium of interest rates as will be discussed in the next section.

III. Results

Exhibit 1 presents the relations between changes in the three-month Treasury bill futures interest rates classified by maturity (n) and unanticipated changes in the Treasury bill spot interest rates. First, the relations between changes in expectations and forecasting errors are consistently positive and high across the maturity of futures contracts: note that β coefficients are significantly greater than zero at any level. Second, the regression coefficients (β) and correlation coefficients (ρ) tend to vary inversely with the maturity of futures contracts (n). These results indicate that expectations on short-term interest rates (three-month Treasury bill futures) are revised substantially on the basis of the forecasting error made within the same time period the revisions take place.

<u>Insert Exhibit 1 about here</u>

Exhibit 2 shows the correlations between changes in expectations with two different maturities: all are very high. For example, the correlation between changes in expectations when n equals 3 and changes in expectations when n = 6 is .976. These high correlations, combined with the results in Exhibit 1, suggest that movements of the yield curve can be described efficiently as a whole in terms of a group of rates each of which is related to the forecasting error and hence to each other systematically.

<u>Insert Exhibit 2 about here</u>

On the other hand, none of the intercept terms of regression equations in Exhibit 1 differs significantly from zero, which has important implications for testing the expectation hypothesis of interest rates. Note that the intercept terms of the regression in the present form may reflect the liquidity or risk premium, as indicated by Meiselman [9]. For example, consider the regression when n equals twenty one months: ${}_{t+21}F - {}_{t+21}F_{t-1} = -.058 + .359\ (R_t - {}_tF_{t-1})$. This regression implies that if the forecasting error $(R_t - {}_tF_{t-1})$ of three-month interest rates equals zero, the market will revise downward expectations of three-month interest rates to prevail twenty one months in the future by .058 percentage units. Alternatively, the regression predicts no change in market expectations of interest rate applicable to twenty one months in the future if the actual three-month interest rate at time t is greater than the futures rate observed at time t-1 by .161 percentage points. In other words, when actual rates are greater than expectations by more than .161 percentage points, expectations will be revised upward on average, and when expectations are less than actual rates by more than .161 percentage points, expectations will be revised downward on average. In this sense, .161 percentage difference can be regarded as a quasi-liquidity premium reflected in the futures contracts on three-month Treasury bills. However, the constant terms of the regression do not significantly differ from zero regardless of n. Thus, this result supports indirectly the expectations hypothesis of the term structure of interest rates asserting that the liquidity or risk premium on default-free discount bonds is zero.[6] It is important to note however that this result may not be an absolute evidence for

the expectation hypothesis because of the simple assumption of the linearity between forecasting errors and changes in expectations. Nevertheless, the intent of this paper is not so much to derive more complex regression equations, as it is to examine the behavior of expectations of interest rates reflected in Treasury bill futures, following the conventional method for simplicity.

Exhibit 3 summarizes the synchronization of three-month Treasury bill rates classified by maturity and unanticipated changes in three-month spot rates. Evidently, the forecasting error and futures rates tend to move in the same direction across n for the sample period: June 1976 - August 1983. For example, when n equal 3, every time the forecasting error was negative, the market revised upward expectations of what interest rates in the future are likely to be (12 out of 12). When the forecasting error was positive (17 times), expectations were revised upward 82.4 percent (14 times out of 17). This result provides another support for the conventional expectations hypothesis.

Insert Exhibit 3 about here

IV. Concluding Remarks

This paper has examined expectations of interest rates contained in three-month Treasury bill futures in a behavioral context. The major findings are as follows: 1) The forecasting error-learning mechanism is a dominant factor for changes in expectations of interest rates reflected in Treasury bill futures contracts, 2) The expectation hypothesis of the term structure of interest rates is indirectly supported in the sense that changes in expectations (reflected in Treasury

bill futures) are significantly synchronized with forecasting errors and that the risk or liquidity premiums do not significantly differ from zero. Even in the absence of the longer maturity Treasury bill futures contracts at the present time, this result suggests at least that the components of the yield curve are members of a group of rates which are systematically related to each other by the forecasting error and the systematic formation of expectations reflected in interest-rate futures.

Footnotes

[1]Thus, we can observe values of market expectations of interest rates directly, instead of using proxies for such expected rates, for example, forward rates which are implicit in the term structure of interest rates.

[2]Currently, futures contracts on other financial securities are available: Treasury bonds, CD, and GNMA. However, there are problems in using these futures as expectations because of the possible biases of futures prices on these securities due to some institutional factors such as timing option, quantity and quality options given to the seller of these futures contracts (see [5] and [7]). This paper employs only three-month Treasury bill futures to avoid this problem as much as possible.

[3]See [2] for details on the Treasury bill futures market.

[4]The MJK computer tape was supplied by MJK Associates in California which is a computer service specializing in the futures markets. All prices are quoted in their normal trading units, as determined by the various exchanges.

[5]As a previous research [9] conjects, "The expectation hypothesis need not be tested by relating yield curves to contemporaneous expectations. Instead, changes in, rather than levels of interest rates can be related to factors which systematically cause revisions of expectations."

[6]The term "indirect" was employed as opposed to the attempt to measure the term structure directly (see [1] and [8]).

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Exhibit 1

Relation between Changes in Treasury Bill Futures Interest Rates and Unanticipated Changes in Spot Interest Rates*

$$_{t+n}F_t - {}_{t+n}F_{t-1} = \hat{\alpha} + \hat{\beta}\,(R_t - {}_tF_{t-1})$$

(units of percentage points)

n (months)	No. of observation	$\hat{\alpha}$	$\hat{\beta}$	ρ	F	$\overline{R}$	D.W.
3	29	-.153 (-1.539)	.718 (8.253)	.946	68.112	.716	2.293
6	29	-.111 (-.896)	.600 (5.499)	.827	30.234	.528	2.208
9	29	-.078 (-.593)	.532 (4.644)	.766	21.563	.444	2.056
12	28	-.120 (-1.045)	.487 (4.925)	.794	24.254	.483	2.391
15	27	-.125 (-1.096)	.452 (4.681)	.783	21.912	.467	2.244
18	24	-.082 (-.678)	.400 (4.077)	.756	16.621	.431	1.908
21	24	-.058 (-.454)	.359 (3.509)	.700	12.315	.359	1.805

*Numbers in parentheses are t values, and notations are as follows:

ρ: Correlation Coefficient

F: 'F' statistics

D.W.: Durbin-Watson statistics.

Exhibit 2

Correlations between Changes in Expectations with Two Different Maturities

$(\Delta_n = {}_{t+n}F_t - {}_{t+n}F_{t-1})$

	Δ_3	Δ_6	Δ_9	Δ_{12}	Δ_{15}	Δ_{18}	Δ_{21}
Δ_3	1.000	.976	.944	.935	.907	.870	.819
Δ_6		1.000	.992	.977	.953	.915	.875
Δ_9			1.000	.994	.978	.946	.916
Δ_{12}				1.000	.994	.972	.946
Δ_{15}					1.000	.991	.971
Δ_{18}						1.000	.992
Δ_{21}							1.000

Exhibit 3

Synchronization of Futures Interest Rate Classified by Maturity and Unanticipated Changes in Spot Rates

1. When the forecasting error is negative ($E_t = R_t - {}_tF_{t-1} < 0$)

Number of times in the future at which futures rate is applicable

n	3	6	9	12	15	18	21	Total
Increased	0	1	2	2	2	2	3	12 (15.4%)
Unchanged	0	1	0	0	1	0	0	2 (2.5%)
Decreased	12	10	10	10	9	7	6	64 (82.1%)
Total	12	12	12	12	12	9	9	78 (100%)

2. When the forecasting error is positive ($E_t = R_t - {}_tF_{t-1} > 0$)

n	3	6	9	12	15	18	21	Total
Increased	14	13	13	12	11	9	9	81 (73.6%)
Unchanged	0	1	0	0	0	1	1	3 (1.8%)
Decreased	3	3	4	4	4	4	4	26 (23.6%)
Total	17	17	17	16	15	14	14	110 (100%)